into two parts, as in (A11),

$$\cdots\int_0^{t-t_1^0} d\tau_2 \int_{V'_{1r,h}} d\omega_1 d\omega_2 = \cdots \int_0^{\tau_c'} d\tau_2 \int_{D'_{r,h}} d\omega_1 d\omega_2 - \cdots \int_0^{\tau_c'} d\tau_2 \int_{D'_{r3,h3}} d\omega_1 d\omega_2, \quad \text{(B6)}$$

where $D'_{r,h}$ denote the regions of τ_2, ω_1, ω_2 which satisfies Eq. (B2), and $D'_{r3,h3}$ denote the regions of τ_2, ω_1, ω_2 which satisfies Eq. (B2) as well as $t_n^*>t$.

Furthermore, we note that the restrictions upon the region of integration in (B5) which come from (B2) with $k\geqslant 4$ will be independent of t_1^0. This is because $\mathbf{R}_{\alpha_k}(k-1)$ and $\mathbf{P}_{\alpha_k}(k-1)$ are independent of t_1^0 [$\mathbf{R}_{\alpha_k}(k-1$, and $\mathbf{P}_{\alpha_k}(k-1)$ only depends upon τ_2, ω_1, and ω_2]. We may, thus, freely integrate the first term on the right-hand side of (B6) over t_1^0 to finally obtain

$$I_{(\alpha_1)\cdots(\alpha_n)}g = [t\Lambda_{(\alpha_1)\cdots(\alpha_n)} + M_{(\alpha_1)\cdots(\alpha_n)}]g, \quad \text{(B7)}$$

where $\Lambda_{(\alpha_1)\cdots(\alpha_n)}$ is the time-independent scattering operator for completed collisions defined by

$$\Lambda_{(\alpha_1)\cdots(\alpha_n)} \equiv \int_0^{\tau_c'} d\tau_2 \int_{D'_{r,h}} d\omega_1 d\omega_2 [C_r' S_r - C_h' S_h] \quad \text{(B8)}$$

and $M_{(\alpha_1)\cdots(\alpha_n)}(t)$ is an operator which corresponds to those initial phase points which lead to incompleted collisions ($t_n^*>t$). This operator is given by

$$M_{(\alpha_1)\cdots(\alpha_n)}(t) \equiv -\int_0^t dt_1^0 \int_0^{\tau'} d\tau_2 \int_{D'_{r3,h3}} d\omega_1 d\omega_2 \times [C_r' S_r - C_h' S_h]. \quad \text{(B9)}$$

The asymptotic time dependence of $M_{(\alpha_1)\cdots(\alpha_n)}(t)$ can be shown to satisfy

$$M_{(\alpha_1)\cdots(\alpha_n)}(t) = O(\ln t) \quad \text{(large } t\text{)}. \quad \text{(B10)}$$

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Coupling-Constant Sum Rules*

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Coupling-constant sum rules are derived assuming that the violation of unitary symmetry transforms like the eighth component of "unitary spin." This parallels the derivation of the mass sum rule.

I. INTRODUCTION

A MODEL of strong interaction symmetry in which both mesons and baryons transform like the eight-dimensional irreducible representation of $SU(3)$ was proposed by Gell-Mann and Nee'man.[1] The immediate prediction of such a model—that the mesons are degenerate and the baryons are degenerate—is clearly false. Nevertheless, the symmetry scheme appears useful.[2] The success of the Gell-Mann mass formula[3] indicates that the breakdown of symmetry occurs in a particularly simple way. In this paper, we apply similar considerations to coupling constants, and we derive coupling-constant sum rules analogous to the Gell-Mann mass formula.

II. COUPLING CONSTANT SUM RULES

The only effective-mass Lagrangian invariant under $SU(3)$ is

$$\mathcal{L}_M = M\,\mathrm{Tr}\bar{B}B, \quad (1)$$

where

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & n \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & p \\ \Xi^0 & \Xi^- & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}. \quad (2)$$

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[3] See also S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 989 (1962); S. Glashow, Phys. Rev. **130**, 2132 (1963), and Ref. 1.

Including possible symmetry-breaking effects, we may write

$$\mathcal{L}_M = m_0\,\mathrm{Tr}\bar{B}B + m_1\,\mathrm{Tr}\bar{B}\lambda_8 B + m_2\,\mathrm{Tr}\bar{B}B\lambda_8 + m_3\,\mathrm{Tr}\bar{B}\lambda_8 B\lambda_8\,, \quad (3)$$

where

$$\lambda_8 = \begin{bmatrix} 1/\sqrt{3} & 0 & 0 \\ 0 & 1/\sqrt{3} & 0 \\ 0 & 0 & -2/\sqrt{3} \end{bmatrix}. \quad (4)$$

Four parameters appear because there are just four isotopic multiplets of baryons. Comparing with the experimental masses, we find

$$\begin{aligned} m_0 &\cong 1150\ \mathrm{MeV}, \\ m_1 &\cong -77\ \mathrm{MeV}, \\ m_2 &\cong 144\ \mathrm{MeV}, \\ m_3 &\cong 3\ \mathrm{MeV}. \end{aligned} \quad (5)$$

The smallness of m_3 suggests that symmetry-breaking effects are predominantly linear in λ_8. Indeed, neglect of the term quadratic in λ_8 ($m_3=0$) gives the Gell-Mann mass formula. That the eightfold way may be a reasonable approximate symmetry scheme is indicated by $m_{1,2}/m_0 \sim \frac{1}{10}$. We now proceed to apply completely analogous reasoning to the form of meson-baryon coupling.

In the eightfold way the interaction Hamiltonian for pseudoscalar Yukawa coupling has the form

$$H_{\mathrm{int}} = g^d\,\mathrm{Tr}(\bar{B}\gamma_5 PB + \bar{B}\gamma_5 BP) + g^f\,\mathrm{Tr}(\bar{B}\gamma_5 PB - \bar{B}\gamma_5 BP)\,, \quad (6)$$

where

$$P = \begin{bmatrix} \pi^0/\sqrt{2} + \chi/\sqrt{6} & \pi^+ & K^0 \\ \pi^- & -\pi^0/\sqrt{2} + \chi/\sqrt{6} & K^+ \\ \bar{K}^0 & K^- & -(2/\sqrt{6})\chi \end{bmatrix}. \quad (7)$$

The g^d term is of the D type under R reflection,[1] while g^f term is of the F type.

Including symmetry breaking effects, we assume that H_{int} (renormalized) takes the form (γ matrices are suppressed)

$$\begin{aligned} H_{\mathrm{int}} = {} & g_0{}^d\,\mathrm{Tr}(\bar{B}PB + \bar{B}BP) + g_0{}^f\,\mathrm{Tr}(\bar{B}PB - \bar{B}BP) \\ & + g_1\,\mathrm{Tr}\bar{B}\lambda_8 PB + g_2\,\mathrm{Tr}\bar{B}\lambda_8 BP + g_3\,\mathrm{Tr}\bar{B}P\lambda_8 B \\ & + g_4\,\mathrm{Tr}\bar{B}B\lambda_8 P + g_5\,\mathrm{Tr}\bar{B}PB\lambda_8 + g_6\,\mathrm{Tr}\bar{B}BP\lambda_8 \\ & + g_7\,\mathrm{Tr}\bar{B}B\,\mathrm{Tr}P\lambda_8 + g_8\,\mathrm{Tr}\bar{B}P\,\mathrm{Tr}B\lambda_8 \\ & + g_9\,\mathrm{Tr}\bar{B}\lambda_8\,\mathrm{Tr}BP\,. \end{aligned} \quad (8)$$

This is the most general pseudoscalar Yukawa interaction that transforms like the eighth and ninth components of unitary spin, akin to Eq. (4) for the effective mass Lagrangian.

Equation (8) involves eleven parameters. We expect only eight parameters in addition to $g_0{}^d$ and $g_0{}^f$ since the one-dimensional representation appears eight times in $8\otimes8\otimes8\otimes8$. Thus, the terms in (8) are not linearly independent. We can verify the identity[4]

$$\begin{aligned} & \mathrm{Tr}\bar{B}\lambda_8 PB + \mathrm{Tr}\bar{B}\lambda_8 BP + \mathrm{Tr}\bar{B}P\lambda_8 B + \mathrm{Tr}\bar{B}B\lambda_8 P \\ & \quad + \mathrm{Tr}\bar{B}PB\lambda_8 + \mathrm{Tr}\bar{B}BP\lambda_8 \\ & = \mathrm{Tr}\bar{B}B\,\mathrm{Tr}P\lambda_8 + \mathrm{Tr}\bar{B}P\,\mathrm{Tr}B\lambda_8 + \mathrm{Tr}\bar{B}\lambda_8\,\mathrm{Tr}BP\,. \end{aligned} \quad (9)$$

By requiring that the interaction Hamiltonian is charge-conjugation invariant, we obtain the restrictions $g_3=g_1$, $g_4=g_6$ and $g_8=g_9$. Using (9) to eliminate the g_4 term, we finally obtain

$$\begin{aligned} H_{\mathrm{int}} = {} & g_0{}^d\,\mathrm{Tr}(\bar{B}PB + \bar{B}BP) + g_0{}^f\,\mathrm{Tr}(\bar{B}PB - \bar{B}BP) \\ & + g_1{}'\,\mathrm{Tr}\bar{B}[\lambda_8, P]B + g_2{}'\,\mathrm{Tr}\bar{B}\lambda_8 BP \\ & + g_5{}'\,\mathrm{Tr}\bar{B}PB\lambda_8 + g_7{}'\,\mathrm{Tr}\bar{B}B\,\mathrm{Tr}P\lambda_8 \\ & + g_8{}'(\mathrm{Tr}\bar{B}P\,\mathrm{Tr}B\lambda_8 + \mathrm{Tr}\bar{B}\lambda_8\,\mathrm{Tr}BP)\,. \end{aligned} \quad (10)$$

There are twelve pseudoscalar coupling constants to be determined between the four baryons N, Σ, Λ, Ξ and the three mesons π, K, χ (the remaining couplings follow from charge independence). We take these twelve constants to be

$$\begin{array}{llll} g_{\bar{n}p\pi^-}, & g_{\bar{\Xi}^+\Xi^0\pi^-}, & g_{\bar{\Sigma}^+\Lambda\pi^-}, & g_{\bar{\Sigma}^+\Sigma^0\pi^-}, \\ g_{\bar{p}p\chi}, & g_{\bar{\Xi}^0\Xi^0\chi}, & g_{\bar{\Sigma}^0\Sigma^0\chi}, & g_{\bar{\Lambda}\Lambda\chi}, \\ g_{\bar{p}\Sigma^0K^+}, & g_{\bar{p}\Lambda K^+}, & g_{\bar{\Sigma}^0\Xi^-K^+}, & g_{\bar{\Lambda}\Xi^-K^+}. \end{array}$$

These 12 coupling constants are described by seven parameters in (10). Thus, five identities among the coupling constants may be deduced. They are

$$g_{\bar{\Lambda}\Xi^-K^+} - g_{\bar{\Sigma}^+\Lambda\pi^-} + \frac{2}{\sqrt{6}} g_{\bar{\Xi}^+\Xi^0\pi^-} + \frac{2}{\sqrt{3}} g_{\bar{p}\Sigma^0K^+} - \frac{1}{\sqrt{3}} g_{\bar{\Sigma}^0\Xi^-K^+} + \frac{\sqrt{3}}{3} g_{\bar{\Sigma}^+\Sigma^0\pi^-} = 0\,, \quad (11a)$$

$$g_{\bar{p}\Lambda K^+} - g_{\bar{\Sigma}^+\Lambda\pi^-} + \frac{2}{\sqrt{6}} g_{\bar{n}p\pi^-} - \frac{1}{\sqrt{3}} g_{\bar{p}\Sigma^0K^+} + \frac{2}{\sqrt{3}} g_{\bar{\Sigma}^0\Xi^-K^+} - \frac{\sqrt{3}}{3} g_{\bar{\Sigma}^+\Sigma^0\pi^-} = 0\,, \quad (11b)$$

$$g_{\bar{\Sigma}^0\Sigma^0\chi} - g_{\bar{\Lambda}\Lambda\chi} - \frac{4}{3\sqrt{6}} g_{\bar{n}p\pi^-} - \frac{4}{3\sqrt{6}} g_{\bar{\Xi}^+\Xi^0\pi^-} - \frac{8}{3\sqrt{3}} g_{\bar{p}\Sigma^0K^+} - \frac{8}{3\sqrt{3}} g_{\bar{\Sigma}^0\Xi^-K^+} + 2 g_{\bar{\Sigma}^+\Lambda\pi^-} = 0\,, \quad (11c)$$

$$g_{\bar{\Sigma}^0\Sigma^0\chi} - g_{\bar{\Xi}^0\Xi^0\chi} + \frac{1}{\sqrt{6}} g_{\bar{\Xi}^+\Xi^0\pi^-} - \frac{4}{\sqrt{3}} g_{\bar{\Sigma}^0\Xi^-K^+} + \frac{1}{\sqrt{3}} g_{\bar{\Sigma}^+\Sigma^0\pi^-} = 0\,, \quad (11d)$$

$$g_{\bar{\Sigma}^0\Sigma^0\chi} - g_{\bar{p}p\chi} + \frac{1}{\sqrt{6}} g_{\bar{n}p\pi^-} - \frac{4}{\sqrt{3}} g_{\bar{p}\Sigma^0K^+} - \frac{1}{\sqrt{3}} g_{\bar{\Sigma}^+\Sigma^0\pi^-} = 0\,. \quad (11e)$$

[4] P. N. Burgoyne (unpublished).

Likewise, we obtain an analogous result for the vector-baryon couplings with the substitution $\pi \to \rho$, $K \to K^*$, $\chi \to \omega$ in (11) since the same $H_{\rm int}$ (10) holds for vector-meson baryon couplings if we let $\gamma_5 \to \gamma_u$ and $P \to V$, where

$$V=\begin{bmatrix} \rho^0/\sqrt{2}+\omega/\sqrt{6} & \rho^+ & K^{0*} \\ \rho^- & -\rho^0/\sqrt{2}+\omega/\sqrt{6} & K^{+*} \\ \bar{K}^{0*} & K^{-*} & -2\omega/\sqrt{6} \end{bmatrix}. \quad (12)$$

We next obtain coupling constant identities for the coupling of a vector meson to two pseudoscalar mesons. Making use of charge conjugation invariance, we get

$$\begin{aligned} H_{\rm int}=g_0{}^f&(\mathrm{Tr}P_{,u}V_uP-P_{,u}PV_u) \\ &+g_1(\mathrm{Tr}P_{,u}\lambda_8V_uP-\mathrm{Tr}P_{,u}PV_u\lambda_8) \\ &+g_2(\mathrm{Tr}P_{,u}\lambda_8PV_u-\mathrm{Tr}P_{,u}V_uP\lambda_8) \\ &+g_3(\mathrm{Tr}P_{,u}V_u\lambda_8P-\mathrm{Tr}P_{,u}P\lambda_8V_u). \quad (13) \end{aligned}$$

Integrating by parts and comparing with (13), we find $g_1=g_3$. There are three parameters and five coupling constants which we take to be $g_{\bar{K}^0K^+\omega}$, $g_{\bar{K}^0K^+\rho^-}$, $g_{\pi^+\pi^0\rho^-}$, $g_{\chi K^-K^{+*}}$, $g_{\pi^-\bar{K}^0K^{+*}}$, so that we obtain the two identities

$$g_{\pi^+\pi^0\rho^-}-(4/\sqrt{2})g_{\pi^-\bar{K}^0K^{+*}}-(1/\sqrt{2})g_{\bar{K}^0K^+\rho^-}+(3/\sqrt{3})g_{K^-K^+\omega}=0, \quad (12a)$$

$$g_{K^-K^+\omega}-g_{\chi K^-K^{+*}}+1/\sqrt{6}g_{\bar{K}^0K^+\rho^-}-1/\sqrt{6}g_{\pi^-\bar{K}^0K^{+*}}=0. \quad (12b)$$

Finally, we obtain identities for the coupling of two vector mesons to a pseudoscalar meson. By making use of charge conjugation invariance and the identity having the form of Eq. (9), we get

$$H_{\rm int}=\epsilon_{\alpha\beta\gamma\delta}\left\{g_0{}^d\,\mathrm{Tr}\frac{\partial V_\alpha}{\partial x_\beta}P\frac{\partial V_\gamma}{\partial x_\delta}+g_1\,\mathrm{Tr}\frac{\partial V_\alpha}{\partial x_\beta}\frac{\partial V_\gamma}{\partial x_\delta}\,\mathrm{Tr}P\lambda_8 +g_2\,\mathrm{Tr}\frac{\partial V_\alpha}{\partial x_\beta}P\,\mathrm{Tr}\frac{\partial V_\gamma}{\partial x_\delta}\lambda_8+g_3\,\mathrm{Tr}\frac{\partial V_\alpha}{\partial x_\beta}\lambda_8\frac{\partial V_\gamma}{\partial x_\delta}P\right\}. \quad (13)$$

The identities are

$$-g_{\rho^+\omega\pi^-}+g_{K^{-*}\omega K^+}+\frac{1}{\sqrt{6}}g_{\rho^-\bar{K}^{0*}K^+}+\frac{2}{\sqrt{6}}g_{\bar{K}^{0*}K^{+*}\pi^-}=0, \quad (14a)$$

$$-2g_{\rho^0\rho^0\chi}+g_{K^{-*}K^{+*}\chi}+\frac{4}{\sqrt{6}}g_{\rho^-\bar{K}^{0*}K^+}-\frac{1}{\sqrt{6}}g_{\bar{K}^{0*}K^{+*}\pi^-}=0, \quad (14b)$$

$$-2g_{\omega\omega\chi}+\frac{2}{3\sqrt{6}}g_{\rho^-\bar{K}^{0*}K^+}+\frac{1}{3\sqrt{6}}g_{\bar{K}^{0*}K^{+*}\pi^-}+g_{K^{-*}K^{+*}\chi}+2g_{K^{-*}\omega K^+}=0, \quad (14c)$$

where, for example, $g_{\rho^+\omega\pi^-}$ is defined to be the coefficient of $(\partial\rho_\alpha{}^+/\partial x_\beta)(\partial\omega_\gamma/\partial x_\delta)\pi^-\epsilon_{\alpha\beta\gamma\delta}$ (the order of factors $\partial\rho/\partial x$, $\partial\omega/\partial x$, π^- being immaterial in the definition) in the expansion of (13).

Equations (11), (12), and (14) are the coupling constant sum rules analogous to the Gell-Mann and Okubo mass formula.

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